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Language ang terms to communicate mathematics

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Popularising mathematics requires a preliminary reflection on language and terms, the choice of which results from underlying dynamics. The aim of this article is to start an overall analysis of the conditions influencing this linguistic choice.

The language of mathematics

In mathematical research the various choices of communication play a crucial role. In the course of time communication has changed, evolved: mathematical results do not become official when they are created and elaborated but rather when they are formalised and presented to the community, i.e. when they are divulged. Most people ignore the fact that Pythagoras' theorem was already centuries-old when Pythagoras or one of his disciples formulated the proposition, because it had been elaborated by Chinese mathematicians ages before the Greek philosopher. Thus the theorem was, is and will always be Pythagoras'.

Sometimes, even the opposite can happen. Pierre de Fermat was the author of an ingenious communicative expedient based on the assertion that “on the contrary, it is impossible for a cube to be written as a sum of two cubes or a fourth power to be written as a sum of two fourth powers or, in general, for any number which is a power greater than the second to be written as a sum of two like powers. I have discovered a truly remarkable proof which this margin is too small to contain.” [5]. That is how a simple, even groundless, conjecture became very famous within the mathematicians’ community known as “Fermat’s last theorem” and how Andrew Wiles’ long and successful commitment to provide a complete demonstration of it did not prove successful enough to change the name of the theorem.

Thus, the creation of the whole so-called *corpus mathematicus* - not only the attribution of some of its milestones – developed along tortuous paths, blind alleys, bends and repetitions. Moreover, the processes of mathematical logic usually follow a path that often divides, through a succession of forks which produce and legitimise a wide range of mathematical notions regardless of unity which became an actual objective only at the beginning of the twentieth century, at the Paris congress, and through the work of the Turin mathematician, Giuseppe Peano.[1]

Peano presented a program of mathematical communication based on four main steps which should shed some light on this discipline both at a horizontal and vertical level.

The scientific community and mathematicians in particular, stated the right/duty to communicate within its boundaries and to overcome those aspects of communication which are usually considered as obstacles in other contexts. This right/duty is applied to horizontal communication which can be considered as being effective whenever it succeeds in establishing a relationship among different scholars regardless of their mother-tongue or institution of affiliation.

On the contrary, vertical communication is a sort of translation of notions and models to the benefit of those who do not belong to a specific scientific community. The purposes of this translation may vary: improving the cultural level of the general public, convincing government and parliament to fund research, increasing the number of mathematically literate to provide new recruits.

Peano’s four steps to standardise the system of mathematical notation can be reinterpreted in the light of the two independent levels of communication: his *Formulario Mathematico*, originally

[1] Cf. [7].

designed to contain all mathematical principles; his proposal for a new mathematical notation (this is Peano's most famous project since his formalism was adopted by Russell and then by the whole scientific community); the proposal of five postulates, such as the Euclidean, to describe the system of natural numbers on an axiomatic basis; the *Latino sine Flexione* and *Interlingua* [6].

The first two steps were elaborated to enable anybody to read and understand mathematics: the *Formulario* was to be the source of knowledge and the system of notation was to provide the basic elements of the mathematical language.

The *Latino sine Flexione* and *Interlingua* was to enable anybody (or at least anybody within the scientific community) to communicate, but these two artificial languages were discarded soon after Peano's death. Nowadays, many years after the first edition of the *Formulario*, the idea of collecting all mathematical principles in one single volume seems rather naive and unfeasible. The mathematical notation is the only part of Peano's unifying project to have been retained by the community since it enabled mathematicians of different epochs and places to communicate, though it also increased the abstractness of mathematical language to the detriment of every layman.

Stella Baruk underscores in the preface of her "*Dizionario di matematica elementare*" (Dictionary of Elementary Mathematics) [3] that "many students still consider mathematics as a foreign, nonsensical language, or rather a sense yet-to-be-grasped. Unlike foreign languages, mathematics seems more complex: there is a formal lack of meaning which may concern the unknown function of a single letter and there is also a basic lack of meaning concerning the fact that mathematics and its language are perceived as uninteresting and unnecessary. These two aspects are not equivalent: the former eventually prevents the solution of the latter". Peano's system of notation was originally elaborated to provide mathematicians of different epochs and places with one single code, but it ended up by becoming a veil which conceals the actual sense of a mathematical proposition to most people.

The fact that there exists an almost unavoidable correspondence between new concepts and new terms aggravates the situation. New terms are based on other pre-existing mathematical terms which are as unfamiliar as the new ones or, when they stem from standard everyday vocabulary, they refer to one concept, and one alone, in a process that excludes any other possible meanings. Mathematical connotations are therefore lacking and consequently many misunderstandings and misinterpretations may result.

The formation of terms can be carried out in many ways such as analogy, fusion, synthesis, division of pre-existing terms and symbols, which were first introduced by Peano in an attempt to formalise all mathematical principles.

For the understanding of mathematics and consequently for the popularisation of this discipline, it is essential to understand its terminology. Essays, books or articles about mathematics contain the explanation (in the sense of disclosure, of complete revelation) of all the terms they are made up of. This should also be the attitude of those who popularise mathematics recalling the typical behaviour of the middle school pupil "who is good at all subjects except maths. He tries to deal with geometry but he becomes depressed. He cannot even draw a figure: mathematical terms are meaningless to him. However he does not give up and gradually moves from "orthocentre" to "height", from "height" to "perpendicular", from "perpendicular" to "right angle" and so on. Then eureka! He draws the figure" [3].

This does not necessarily imply that all the steps of a mathematical theory should be explained and retraced, otherwise it would be virtually impossible to popularise more complicated concepts such as fractals, variations and surface topology. Dealing exhaustively with every passage of the development of a mathematical theory would make the whole explanation meaningless to most.

Meeting the reader's expectations is extremely important in divulging mathematics. Each layman expects mathematics to be cohesive and coherent, that is why reference to anything non-contextual is considered vexing. If the reasoning is based on acts of faith, there is no valid explanation.

Sometimes, however, acts of faith are necessary and every layman ends up believing in something about which he/she is totally ignorant. It is clearly up to the author of the paper to deal with the topic exhaustively by choosing to avoid generalisations and privileging a single aspect, a clarifying example. The actual goal of communicating mathematics is not to illustrate the complete development of a theory, but rather to convey the underlying idea or process and to underscore the exactness and method used to develop it. The author should state clearly that he/she is dealing with one single aspect of the whole topic in order to avoid meaningless generalisations.

A simplified language is crucial to the popularisation of mathematics: technical terms should perhaps be completely abolished. A mathematical term is usually monosemic but, if the receiver of the message is unaware of the meaning of that term, the whole paper loses its rigour and becomes vague. It assumes different shades of meaning that do not exist in mathematics which confuse and dilute that rigour essential to the development of logical reasoning.

The diamond merchants

Robin Dunbar writes in his book: "the diamond merchants of New York and Amsterdam are archetypal examples of how a trading community works. A man's word is his guarantee since everybody knows about him, his honesty and trustworthiness. The merchants of gemstones form a small, enclave made up of familiar faces and personal relationships. Neither contracts nor written documents are required since everything is based on trust. This is only possible in that the community is small. This system would collapse if more people were allowed to be part of it" [4]. At a horizontal level, communication among mathematicians does not differ from communication among diamond merchants, since also the mathematicians form a restricted community which, for more than a century, has split into countless smaller enclaves that can hardly communicate between each other. The type of communication between two of these enclaves is very similar to "vertical communication"; the only difference being that the mathematicians share the same language and system of notation.

Within each, restricted world, there are norms that allow rapid and effective communication. People easily resort to *ipse dixit* since it is absolutely normal for a logical passage, a demonstration or even a whole theory to be taken for granted in the name of its authoritative source. People also resort to informal mathematics whereas the institutional sources such as lectures, meetings, articles, proceedings, pre-prints and the like formalise the results. The origin of the latter, summed up in Galileo's motto "try and try again", can be identified in informal chats taking place sometimes in front of a blackboard, more frequently in the corridors of a department and more often with a cup of coffee. The production of new mathematical knowledge is therefore based on mutual trust, direct relationship, friendliness and the use of a common, basic jargon. Each mathematician resorts to his own poor and superficial knowledge of English for scientific purposes which is made up of an untranslatable jargon – it is usually clumsily adapted to the different national languages – and an inevitably scant vocabulary.

More specifically, the possible reason for the use of this poor jargon is that ordinary people generally deal with different topics using the most suitable language spontaneously [10]. Each individual, from housewife to experts, acquires spontaneously and more or less consciously, the language of his/her specific field of activity. This language rapidly becomes an irreplaceable means of communication and unambiguous exchange of information. Unlike ordinary, everyday language, each term of a specialised area is monosemic, monoreferential, specific, related to other terms and collocation-bound, thus creating a more rigid linguistic system. In a technical or scientific context the necessity to adopt these terms is stringent: its aim is to enable the receiver of the message to acquire rapidly the new terminology [10].

These communicative habits continue because mathematicians' communities are usually very small and yet open to new members, but they also hamper each attempt to take information outside these small communities.

It may be pertinent to reinterpret Dunbar's view in mathematical terms "the small world [of diamond merchants] can be compared to the amorphous maxi-networks in the international financial market. Thousands of people usually unrelated to one another are connected all over the world by means of the new technology. To what extend is the current chaos of financial and insurance markets a consequence of their own size? Dishonest peddlers can often get away scott-free because they work in a big, anonymous market where no trust or duty are expected. There is however a small part of them who are still convinced that they live in those small communities of the past where trade was based on personal trust. In modern electronic market, traders and customers do not know each other. As mutual trust among strangers is rather fragile, current behaviours will rapidly evolve into new and less practical standards" [4]. Mathematicians feel the same sense of hostile distance when they try to communicate the results of their research at a vertical level. They cannot avoid running into the rest of the world which is actually very big and very vast and that consequently does not recognise the authority of the same referents; does not share the same jargon and does not rely on mutual trust.

Vertical communication necessarily implies giving up the above mentioned habits because, though they help communication and understanding at a horizontal level, they hinder vertical exchange of information.

It is also true that, with very few exceptions, those who are professional mathematicians cannot communicate their knowledge at a vertical level. This task should be performed by those who can explain basic mathematical notions in a comprehensible way, by means of meaningful illustrations and, above all, without resorting to *ipse dixit*, informal mathematics, mutual trust and jargon.

Normally mathematics unfolds down lengthy pathways along which rapid and safe short-cuts are allowed, but these have to be explained to the layman to reveal their essence.

New terms and expressions coined to popularise mathematics must replace the power of language, be it symbolic or otherwise. This new terminology, however, should always respect the original unambiguousness of each mathematical term.

People are often unable "to talk" about mathematics because it is confusing and leads to the use of some automatisms to the detriment of real meaning. People cannot forget the sense of helplessness, anguish and resignation it caused them in school, therefore it is also necessary to help re-establish a new, positive attitude towards this discipline.

Thus, the first step towards the popularisation of mathematics is the introduction of a brand-new terminology which keeps its traditional rigour and precision.

It is not simply a question of creating new terminology for an effective popularisation of mathematics, rather, it is a problem of starting a general debate on terminology in the light of the relationship between the general public and the topic to be divulged.

Faith, trust and sources

Three of the four usual ways of effectively communicating mathematics at a horizontal level are based on trust: trust in authority – *ipse dixit* -, in personal relationships and in informal and occasional exchange of information. Although these aspects are only marginal in this specific context, they cannot be totally ignored.

First of all, it is necessary to underscore again the fact that mathematicians form a small universe in which, as happens to the diamond merchants, trust plays a leading and vital role. Consequently when they deal with vertical communication they feel betrayed by the public's not sharing the same communicative automatisms.

Secondly, there is the problem of sources. The fact that people do not share the same cultural background can actually cause confusion.

Thirdly, the absence of informal mathematics can also cause a sense of displacement. That is why some of those who popularise mathematics (there are also some mathematicians among them) have adopted different approaches: there are those who choose the "variety actor" approach and try to attract people with brilliant solutions; there are those who prefer the "lecture-like" dimension and others who cannot help resorting to jokes and funny stories that are actually hardly understandable by ordinary people.

It must be underlined also that the most successful attempts to popularise this discipline usually take place in informal contexts. This is not a necessary or sufficient guarantee to success but it can help mathematics and the public come closer.

The difficult approach to the problem of trust also concerns the general public's point of view.

On the one hand, there is the public's need for stable reference points, such as experts who are still considered trustworthy even when they talk about facts that are not within their competence. On the other hand, the public is often convinced that those who are good at mathematics must necessarily be more clever and gifted than the rest of us. This approach discourages people from believing themselves capable of understanding this discipline.

From the layman's point of view too, *ipse dixit* is therefore both a necessity and an obstacle. In the case of advanced mathematical issues, everybody is willing to accept certain concepts as acts of faith. It is generally accepted that an expert may refer to concepts and notions that he takes for granted. There is only a very small difference between a careful use of *ipse dixit* and an excessive use of it which can jeopardise any attempt to understand the discipline.

In the same perspective, informal communication, particularly in the case of mathematics, can help avoid the sense of helplessness, anguish and resignation which was felt by the laymen of the past.

In the attempt to bring both points of view together, it can be said that "trust" is still a major problem. In vertical communication it upsets a limited universe in which reliable references are uncommon. Informal communication, however, can perform an useful function: from the mathematicians' point of view, it recreates the informal environment of informal mathematics whereas, from the layman's point of view, it helps knock mathematics off its pedestal. Furthermore, the mathematician who decides to communicate at an informal level, demonstrates that he/she trusts the public who, in return, interacts and responds, though maintaining an inborn faith in those who can deal with such difficult notions. This eventually creates the conditions for scientists and the public to meet half-way.

Pidgin mathematical English

Popularising mathematics and science in general implies tackling the question of Italian translation of scientific texts. The presence of many foreign or mistranslated terms is a consequence of bad habit and laziness on the part of mathematicians and hinders the popularisation of science and its communication between various sectors.

English is the language of science, as happened in the past with other languages in different contexts – Italian in music, Latin in law, French in diplomacy and so on – that is why science magazines are written in English and pidgin English is spoken among students and researchers. As a result, ordinary people are excluded from this kind of communication and a linguistic problem is added to conceptual difficulties.

Besides, if all Italians could speak better English as is the case in northern European countries, this language would still be considered as the vehicular language of mathematics and science. Nowadays, scientific translation is mainly a matter of linguistic creation and it plays a fundamental role in popularising science. The creation of an Italian mathematical terminology should be given priority in the production and popularisation of mathematics.

Is it true, as maintained by mathematicians and scientists, that mathematics and science can develop even if they cannot be effectively divulged in Italian? Assuming that mathematics is part of a nation's culture, can such an important part lack the linguistic tools for its expression?

The current situation is that spoken Italian tends not to absorb the specific language of mathematics, not even in the form of linguistic loans. In the past "most ancient linguistic loans were adapted to the receiving language, whereas modern loans are not altered at all" [1]. On the contrary, English can easily retain foreign words and expressions and that is the main reason why it has become the vehicular language of science.

From the point of view of the whole international mathematicians' community, the almost exclusive use of English at a horizontal level impoverishes also internal communication; it is actually a superficial usage which relies on a non-specific, small terminology database. It is also deprived of those linguistic nuances which make up a modern language and can produce effective communication rich in specific terms and poor in common words.

The frequent usage of English terms in the world's communication of science, both at a vertical and horizontal level, often leads to a loss of information. The expression "electron spin", for example, may remind English-speaking people of something they already know, whereas for an Italian speaker it is a purely conventional symbol.

Therefore, on the one hand mathematics and science tend to adopt a technical, though poor language, on the other national languages gradually lose their ability to convey information. Thus Italian mathematicians, for example, may be considered as partially responsible for the decadence of the standard language. According to Tullio De Mauro, this decadence results also from "the general opinion of most people, not only of literati, that specialised languages are separate from the rest, that they do not belong to everyday usage" [1].

Communication experts should therefore co-operate with linguists and translators to develop a new and accessible terminology to be also used in the communication of mathematics; otherwise linguistic problems will always be added to conceptual difficulties.

Five strategies of mathematical communication

In the documentary film of the "Horizon" series broadcast by the BBC under the title "Fermat's Last Theorem", Simon Singh resorts to an effective method to explain the value of Taniyama-Shimura's conjecture. This conjecture, having as a corollary Fermat's last theorem, was eventually demonstrated by Andrew Wiles and provides a natural link between two seemingly distant mathematical theories, namely elliptic equations and modular forms. The parts of the documentary concerning the conjecture are associated by Singh with sights of the Golden Gate or some other famous bridges. The audience's impression is thus one of "linking", which continues even when they are not able to catch the mathematical meaning of what is being said. The bridge – a much more concrete and tangible object than Fermat's last theorem or Taniyama-Shimura's conjecture – becomes the symbol of the link. Singh, like all the authors of the BBC "Horizon" series, pays utmost attention to the language of words, sounds and images, which he uses to weave threads complementing and enhancing the original message. The technique of repetition fixes the meaning, makes it univocal and replaces with effectiveness the power and rigour of a definition otherwise difficult to recreate. The same applies to the frequent use of words like “therefore”, “thus”, “moreover”, etc., which may help reproduce a demonstration-like context.

Focussing one’s attention on terminological choices, there is no need to change names as Hans Magnus Enzensberger does in “The Number Devil”. Why should “unreasonable numbers” be more meaningful than “irrational numbers”? It would be more useful to reduce technical terms in the text. Indeed, it may be better to deal with an issue or discuss an idea without resorting to specific terminology. Why talk about “smooth curve”, when it would be clearer to describe it as “a curve admitting a tangent line”? The first case implies an explanation of the geometric situation – the existence of a tangent line at each point of the curve – and its association to the proper nomenclature. In the second case, the focus is on the meaning. The sense of helplessness, anguish and resignation felt by those who ceased to practice mathematics after the end of school is mainly caused by the difficulty of its system of notation and terminology. Often, to these people a mathematical term is nothing but merely a conventional symbol, lacking any kind of content. Changing this situation and explaining meanings without a signifier might be instructive and clarifying, as it would overcome one of the main obstacles to comprehension.

A further possibility consists in resorting to mathematical analogies related to facts and concepts which are presumably known. As in the case of highly symbolic images, such as Simon Singh’s bridge, these analogies recall similar concepts which are already familiar. To prove effective, an analogy must resort to contextual or well-established concepts with which the reader is certainly acquainted.

Another device in vertical communication is the expression of judgements. This practice, which is virtually non-existent in horizontal communication, may prove a very useful tool in drawing the reader’s attention to a certain fact. Saying that the “zero product” property is a powerful instrument is not the same as saying that it is the key to all solutions, as it reduces the difficulty of all equations. Even if a little exaggeration is allowed, it is sufficient not to lie (indeed, some equations may not be simplified, but certainly none can be complicated), the cost of a little exaggeration is certainly balanced by the possibility to focus the public’s attention.

Many other communicative devices and strategies could be analysed with a view to mathematical communication. However, the five methods which have been mentioned here need to pondered further and improved: symbolic images provide readers with a non-mathematical *fil rouge* enabling them to understand even the most cryptic passages; repetitions may effectively replace definitions; the reduction of specific terminology brings down psychological barriers; mathematical analogies create a sense of familiarity and unity of the concepts being expressed, and the exaggeration of some aspects helps focus the reader's attention on fundamental points.

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